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D-branes in N=2 WZW models

Ulf Lindström[a1] and **Maxim Zabzine**[b2]

[a]*Department of Theoretical Physics,*
Uppsala University, Box 803, SE-751 08 Uppsala, Sweden

[b]*INFN Sezione di Firenze, Dipartimento di Fisica*
Via Sansone 1, I-50019 Sesto F.no (FI), Italy

ABSTRACT

We briefly review the construction of N=2 WZW models in terms of Manin triples. We analyse the restrictions which should be imposed on the gluing conditions of the affine currents in order to preserve half of the bulk supersymmetry. In analogy with the Kähler case there are two types of D-branes, A- and B-types which have a nice algebraic interpretation in terms of the Manin triple.

[1]e-mail address: ulf.lindstrom@teorfys.uu.se
[2]e-mail address: zabzine@fi.infn.it

1 Introduction

During the last few years D-branes on group manifolds have received a great deal of attention (see, e.g., [1] and the references therein). Using methods from CFT one obtains a microscopic decription of various D-branes on group manifolds in terms of conformally invariant boundary states. Therefore the group manifolds provide an ideal laboratory for the study of quantum D-branes on general curved backgrounds.

Recently, the study of boundary conditions that preserve various symmetries on the boundary of sigma models representing open strings [2]-[6] has revealed a close connection to the geometry of the targetspace. The present letter may be viewed as an application of results from those studies.

Here we study D-branes on group manifolds which preserve a certain amount of world-sheet supersymmetry, in particular we focus on the N=2 case. It turns out that for the WZW model N=2 supersymmetry alone imposes strong restrictions on the possible gluing conditions of the affine currents and that these restrictions have a nice algebraic description. The study in this letter is confined to the case where the gluing conditions imposed on the affine currents are given in terms of a constant map $R^A_{\ B}$. Although this restriction is not necessary, the case of constant $R^A_{\ B}$ is the most interesting from the CFT point of view. In this study we apply our previous results [5] for D-branes of general N=(2,2) supersymmetric sigma model to the WZW model.

The letter is organised as follows. In Section 2 we review the N=(2,2) supersymmetry for the WZW models and briefly explain how N=2 supersymmetry is related to the Manin triple $(\mathbf{g}, \mathbf{g}_-, \mathbf{g}_+)$. In Section 3 using the N=2 on-shell transformations we study the requirements which need to be imposed on the gluing conditions of the affine currents to preserve half of the world-sheet supersymmetry. In analogy with the Kähler case we introduce two type of branes: A-type and B-type. Both types of branes have an algebraic description in terms of the Manin triple. In Section 4 we analyse the N=2 superconformal boundary conditions by imposing boundary conditions on the N=2 currents. Finally, in Section 5, we summarize the results and give some examples.

2 N=2 WZW models

First, let us recall the description of N=2 supersymmetry for non-linear sigma models with torsion [7] (for a recent discussion see [8]). The N=1 superfield bulk action for the real scalar superfields Φ^μ is

$$S = \int d^2\sigma\, d^2\theta\ D_+\Phi^\mu D_-\Phi^\nu (g_{\mu\nu}(\Phi) + B_{\mu\nu}(\Phi)), \tag{2.1}$$

where we assume that $H \equiv dB \neq 0$. This action is manifestly supersymmetric under one supersymmetry because of its N=1 superfield form. Further (2.1) admits an additional nonmanifest supersymmetry of the form

$$\delta_2\Phi^\mu = \epsilon_2^+ D_+\Phi^\nu J^\mu_{+\nu}(\Phi) + \epsilon_2^- D_-\Phi^\nu J^\mu_{-\nu}(\Phi). \tag{2.2}$$

Classically this ansatz is unique for dimensional reason. The standard on-shell N=2 supersymmetry requires $J^\mu_{\pm\nu}$ to be complex structures, i.e.

$$J^\mu_{\pm\lambda} J^\lambda_{\pm\nu} = -\delta^\mu_{\ \nu} \tag{2.3}$$

and

$$\mathcal{N}^\rho_{\ \mu\nu}(J_\pm) = J^\gamma_{\pm\mu}\partial_{[\gamma} J^\rho_{\pm\nu]} - J^\gamma_{\pm\nu}\partial_{[\gamma} J^\rho_{\pm\mu]} = 0. \tag{2.4}$$

Invariance of the action (2.1) under the transformations (2.2) requires the metric $g_{\mu\nu}$ to be Hermitian with respect to both complex structures

$$J^\mu_{\pm\rho} g_{\mu\nu} J^\nu_{\pm\lambda} = g_{\rho\lambda} \tag{2.5}$$

and the complex structures to be covariantly constant with respect to different connections

$$\nabla^{(\pm)}_\rho J^\mu_{\pm\nu} \equiv J^\mu_{\pm\nu,\rho} + \Gamma^{\pm\mu}_{\ \ \rho\sigma} J^\sigma_{\pm\nu} - \Gamma^{\pm\sigma}_{\ \ \rho\nu} J^\mu_{\pm\sigma} = 0, \tag{2.6}$$

where we have defined the two affine connections as

$$\Gamma^{\pm\mu}_{\ \ \rho\nu} = \Gamma^\mu_{\ \rho\nu} \pm g^{\mu\sigma} H_{\sigma\rho\nu}, \tag{2.7}$$

with $\Gamma^\mu_{\ \rho\nu}$ the Christoffel connection for the metric $g_{\mu\nu}$. Using (2.6) the inegrability condition (2.4) may be rewritten in alternative form

$$H_{\delta\nu\lambda} = J^\sigma_{\pm\delta} J^\rho_{\pm\nu} H_{\sigma\rho\lambda} + J^\sigma_{\pm\lambda} J^\rho_{\pm\delta} H_{\sigma\rho\nu} + J^\sigma_{\pm\nu} J^\rho_{\pm\lambda} H_{\sigma\rho\delta}. \tag{2.8}$$

To summarize, the transformation (2.2) is a second supersymmetry provided that (2.3), (2.5), (2.6) and (2.8) are satisfied.

We now turn to N=2 supersymmetric WZW models. The WZW models represent a special class of non-linear sigma models defined over a group manifold $\mathcal{M}$ of some Lie group $\mathcal{G}$. The isometry group, $\mathcal{G} \times \mathcal{G}$ is generated by the left and right invariant Killing vectors l^μ_A and r^μ_A respectively, where $A = 1, 2, ..., \dim \mathcal{G}$. They satisfy

$$\{l_A, l_B\} = f_{AB}^{\ \ \ C} l_C, \qquad \{r_A, r_B\} = -f_{AB}^{\ \ \ C} r_C, \qquad \{l_A, r_B\} = 0, \tag{2.9}$$

where $\{,\}$ is the Lie bracket for the vector fields. We restrict ourselves to semi-simple Lie groups, so that the Cartan-Killing metric η_{AB} has an inverse η^{AB} and can be used to raise and lower Lie algebra indices. Both l^μ_A and r^μ_A can be regarded as veilbeins, with inverses l^A_μ, r^A_μ respectively. To define the sigma model, we choose the invariant metric

$$g_{\mu\nu} = \frac{1}{\rho^2}\, l^A_\mu l^B_\nu \eta_{AB} = \frac{1}{\rho^2}\, r^A_\mu r^B_\nu \eta_{AB} \tag{2.10}$$

while $H_{\mu\nu\rho}$ is proportional to the structure constants of the corresponding Lie algebra $\mathbf{g}$

$$H_{\mu\nu\rho} = \frac{1}{2} k\, l^A_\mu l^B_\nu l^C_\rho f_{ABC} = \frac{1}{2} k\, r^A_\mu r^B_\nu r^C_\rho f_{ABC} \tag{2.11}$$

and where ρ and k are constants and k must satisfy a quantization condition. If $\rho^2 = \pm 1/k$ then $H_{\mu\nu\rho}$ is the parallelizing torsion on the group manifold and this is also precisely the relation bewteen the coupling constants that holds at the conformally invariant fixed-point of the beta-functions. Since we are interested in the conformal model, in the following discussion we set $\rho^2 = 1/k$ and $k = 1$ because in our calculations k appears only as overall factor. We are thus interested in the sigma model (2.1) with $g_{\mu\nu}$ and $H_{\mu\nu\rho}$ given by (2.10) and (2.11). Using the above properties we see that the left and right invariant Killing vectors satisfy the following equations

$$\nabla^{(-)}_\rho l^\mu_A = 0, \qquad \nabla^{(+)}_\rho r^\mu_A = 0, \tag{2.12}$$

where $\nabla^{(\pm)}_\rho$ are the affine connections defined in (2.7). Due to (2.12) there are chiral (antichiral) Lie algebra valued currents

$$\mathcal{J}^A_- = l^A_\mu D_- \Phi^\mu, \qquad \mathcal{J}^A_+ = r^A_\mu D_+ \Phi^\mu, \tag{2.13}$$

such that $D_\mp \mathcal{J}^A_\pm = 0$. The components of these currents are defined as follows

$$j^A_\pm = \mathcal{J}^A_\pm|, \qquad k^A_{\substack{+\\=}} = -iD_\pm \mathcal{J}^A_\pm|. \tag{2.14}$$

Instead of using coordinates Φ^μ, the group manifold can be parametrized by group elements in some representation with the generators T_A satisfying

$$[T_A, T_B] = f_{AB}{}^C T_C, \qquad tr(T_A T_B) = -\kappa \eta_{AB}. \tag{2.15}$$

However in this letter we will not use the parametrization in terms of group elements.

The problem of N=2 supersymmetry for the WZW models was first addressed in [9]-[11]. However, we will not follow the original presentation.

Let us assume that the complex structures in (2.2) have the form

$$J^\mu_{-\nu} = l^\mu_A J^A{}_B l^B_\nu, \qquad J^\mu_{+\nu} = r^\mu_A J^A{}_B r^B_\nu \tag{2.16}$$

where J^A_B is a constant matrix acting on the Lie algebra. The relations (2.6) are then automatically satisfied. The remaining properties (2.3), (2.5) and (2.8) may be rewritten in terms of J^A_B as follows

$$J^A_C J^C_B = -\delta^A_B \tag{2.17}$$

$$J^C_A \eta_{CD} J^D_B = \eta_{AB} \tag{2.18}$$

$$f_{ABC} = J^D_A J^L_B f_{DLC} + J^D_B J^L_C f_{DLA} + J^D_C J^L_A f_{DLB} \tag{2.19}$$

Thus we have to construct a J^A_B on the Lie algebra $\mathbf{g}$ with properties (2.17)-(2.19). That is possible only for even dimensional Lie algebras. J^A_B has as eigenvalues $\pm i$ and we choose a basis $T_A = (T_a, T_{\bar{a}})$ on the Lie algebra $\mathbf{g}$ such that J^A_B is diagonal: $J^a_b = i\delta^a_b$, $J^{\bar{a}}_{\bar{b}} = -i\delta^{\bar{a}}_{\bar{b}}$. In this basis eq. (2.18) leads to $\eta_{ab} = \eta_{\bar{a}\bar{b}} = 0$ and the (2.19) gives $f_{abc} = f_{\bar{a}\bar{b}\bar{c}} = 0$. Taken together, this implies that $f_{ab}{}^{\bar{c}} = 0$ and $f_{\bar{a}\bar{b}}{}^{c} = 0$. Therefore $\{T_a\}$ and $\{T_{\bar{a}}\}$ form Lie subalgebras $\mathbf{g}_+$ and $\mathbf{g}_-$ correspondingly. These subalgebras are also maximally isotropic subspaces with the respect to η. Thus the complex structures on the even dimensional group is related to a decomposition of Lie algebra $\mathbf{g}$ into two maximally isotropic sublagebras with respect to η such that $\mathbf{g} = \mathbf{g}_- \oplus \mathbf{g}_+$ as a vector space. Such a structure is called a Manin triple $(\mathbf{g}, \mathbf{g}_-, \mathbf{g}_+)$ and was initially introduced by Drinfeld in the context of completly integrable systems and quantum groups [12]. The relevance of the Manin triples to the N=2 supersymmetry on the group manifolds was pointed out in [13].

In the general situation the relation (2.16) should be replaced by the following

$$J^\mu_{-\nu} = l^\mu_A J^A_B l^B_\nu, \qquad J^\mu_{+\nu} = r^\mu_A \tilde{J}^A_B r^B_\nu \tag{2.20}$$

where J^A_B and $\tilde{J}^A_B$ should each satisfy the relations (2.17)-(2.19) and thus they should be identified with different Manin triples. Therefore, in the general case, the left and right supersymmetries may correspond to different Manin triples (however with respect to the same ad-invariant bilinear nondegenerate form η). Here we only consider the situation where left and right supersymmetries correspond to the same Manin triple.

3 N=2 boundary conditions

In this section we discuss the boundary conditions of N=2 WZW models from the point of view of N=2 supersymmetry. In the next section we address the N=2 superconformal boundary conditions.

Since the classical model does not have a dimensionful parameter the most general local classical boundary condition for the fermions is given by the following expression[3] [5]

$$\psi_-^\mu = \eta_1 R^\mu_{\ \nu}(X)\psi_+^\nu \tag{3.1}$$

In terms of the fermionic component (2.14) of the N=1 affine currents, equation (3.1) can be rewritten as

$$j_-^A = \eta_1 R^A_{\ B}(X) j_+^B \tag{3.2}$$

where $R^\mu_{\ \nu} = l^\mu_A R^A_{\ B} r^B_{\ \nu}$. In what follows we focus on the case when $R^A_{\ B}$ is independent of X. This is the case usually considered in the study of boundary CFT.

We want to understand what kind of restrictions should be imposed on $R^A_{\ B}$ for N=2 supersymmetry to be preserved. In components the manifest on-shell supersymmetry transformations are

$$\left\{ \begin{array}{l} \delta_1 X^\mu = -(\epsilon_1^+ \psi_+^\mu + \epsilon_1^- \psi_-^\mu) \\ \delta_1 \psi_+^\mu = -i\epsilon_1^+ \partial_{\mp} X^\mu + \epsilon_1^- \Gamma^{-\mu}_{\ \nu\rho} \psi_-^\rho \psi_+^\nu \\ \delta_1 \psi_-^\mu = -i\epsilon_1^- \partial_{=} X^\mu - \epsilon_1^+ \Gamma^{-\mu}_{\ \nu\rho} \psi_-^\rho \psi_+^\nu \end{array} \right. \tag{3.3}$$

and the nonmanifest supersymmetry transformations (2.2) are

$$\left\{ \begin{array}{l} \delta_2 X^\mu = \epsilon_2^+ \psi_+^\nu J^\mu_{+\nu} + \epsilon_2^- \psi_-^\nu J^\mu_{-\nu} \\ \delta_2 \psi_+^\mu = -i\epsilon_2^+ \partial_{\mp} X^\nu J^\mu_{+\nu} - \epsilon_2^- J^\mu_{-\sigma} \Gamma^{-\sigma}_{\ \nu\rho} \psi_-^\rho \psi_+^\nu + \epsilon_2^+ J^\mu_{+\nu,\rho} \psi_+^\nu \psi_+^\rho + \epsilon_2^- J^\mu_{-\nu,\rho} \psi_-^\nu \psi_+^\rho \\ \delta_2 \psi_-^\mu = -i\epsilon_2^- \partial_{=} X^\nu J^\mu_{-\nu} + \epsilon_2^+ J^\mu_{+\sigma} \Gamma^{-\sigma}_{\ \nu\rho} \psi_-^\rho \psi_+^\nu + \epsilon_2^+ J^\mu_{+\nu,\rho} \psi_+^\nu \psi_-^\rho + \epsilon_2^- J^\mu_{-\nu,\rho} \psi_-^\nu \psi_-^\rho . \end{array} \right. \tag{3.4}$$

Starting from the fermionic ansatz (3.1) and applying both supersymmetry transformations, (3.3) and (3.4) we should get the bosonic boundary conditions. The result of the first transformation is

$$\partial_{=} X^\mu - R^\mu_{\ \nu} \partial_{\mp} X^\nu + 2i(P^\sigma_{\ \gamma} \nabla_\sigma R^\mu_{\ \nu} + P^\mu_{\ \rho} g^{\rho\delta} H_{\delta\sigma\gamma} R^\sigma_{\ \nu}) \psi_+^\gamma \psi_+^\nu = 0 \tag{3.5}$$

where $\epsilon_1^+ = \eta_1 \epsilon_1^-$. The second supersymmetry gives

$$\begin{array}{l} \partial_{=} X^\mu + (\eta_1\eta_2) J^\mu_{-\lambda} R^\lambda_{\ \sigma} J^\sigma_{+\nu} \partial_{\mp} X^\nu + i \Big[(\eta_1\eta_2) J^\mu_{-\lambda} \nabla^{(-)}_\rho R^\mu_{\ \nu} J^\rho_{+\gamma} + \\ +(\eta_1\eta_2) J^\mu_{-\lambda} R^\lambda_{\ \sigma} J^\sigma_{+\rho} H^\rho_{\ \nu\gamma} + J^\mu_{-\lambda} \nabla^{(+)}_\rho R^\lambda_{\ \nu} J^\rho_{-\sigma} R^\sigma_{\ \gamma} - H^\mu_{\ \rho\sigma} R^\sigma_{\ \gamma} R^\rho_{\ \nu} \Big] \psi_+^\gamma \psi_+^\nu = 0 \end{array} \tag{3.6}$$

where $\epsilon_2^+ = \eta_2 \epsilon_2^-$ and we have used the property (2.6). The boundary conditions (3.5) and (3.6) should be equivalent. Starting from the X-part we get the condition

$$J^\mu_{-\nu} R^\nu_{\ \lambda} = (\eta_1\eta_2) R^\mu_{\ \nu} J^\nu_{+\lambda}. \tag{3.7}$$

In analogy with the Kähler case we use the notation "A-type" condition when $\eta_1\eta_2 = -1$ and "B-type" when $\eta_1\eta_2 = 1$.

[3]We found it convenient to introduce the parameter η_1 which takes on the values ± 1 and correspond to the choice of spin structure.

Using the (3.7) the equation (3.6) is rewritten as

$$\partial_{=}X^{\mu} - R^{\mu}_{\ \nu}\partial_{+\!\!+}X^{\nu} + i\Big[(\eta_1\eta_2)J^{\mu}_{-\lambda}\nabla^{(-)}_{\rho}R^{\mu}_{\ \nu}J^{\rho}_{+\gamma}+ \\ +(\eta_1\eta_2)J^{\mu}_{-\lambda}\nabla^{(+)}_{\rho}R^{\lambda}_{\ \nu}R^{\rho}_{\ \sigma}J^{\sigma}_{+\gamma} - R^{\mu}_{\ \lambda}H^{\lambda}_{\ \nu\gamma} - H^{\mu}_{\ \rho\sigma}R^{\sigma}_{\ \gamma}R^{\rho}_{\ \nu}\Big]\psi^{\gamma}_{+}\psi^{\nu}_{+} = 0 \tag{3.8}$$

Using (2.8) we further rewrite (3.8) as

$$\partial_{=}X^{\mu} - R^{\mu}_{\ \nu}\partial_{+\!\!+}X^{\nu} + 2iJ^{\sigma}_{+\gamma}J^{\lambda}_{+\nu}\left(P^{\rho}_{\ \sigma}\nabla_{\rho}R^{\mu}_{\ \lambda} + P^{\mu}_{\ \phi}H^{\phi}_{\ \rho\sigma}R^{\rho}_{\ \lambda}\right)\psi^{\gamma}_{+}\psi^{\nu}_{+} = 0 \tag{3.9}$$

Comparing the two-fermion terms of (3.5) and (3.9) we get

$$P^{\sigma}_{\ [\gamma|}\nabla_{\sigma}R^{\mu}_{\ |\nu]} + P^{\mu}_{\ \rho}H_{\rho\sigma[\gamma}R^{\sigma}_{\ \nu]} - J^{\sigma}_{+[\gamma}J^{\lambda}_{+\nu]}\left(P^{\rho}_{\ \sigma}\nabla_{\rho}R^{\mu}_{\ \lambda} + P^{\mu}_{\ \phi}H^{\phi}_{\ \rho\sigma}R^{\rho}_{\ \lambda}\right) = 0. \tag{3.10}$$

Using the projectors $\Omega^{+}_{\pm} = 1/2(I \pm iJ_{+})$ we rewrite the above condition as

$$\Omega^{+\sigma}_{\pm[\gamma}\Omega^{+\lambda}_{\pm\nu]}\left(P^{\rho}_{\ \sigma}\nabla_{\rho}R^{\mu}_{\ \lambda} + P^{\mu}_{\ \phi}H^{\phi}_{\ \rho\sigma}R^{\rho}_{\ \lambda}\right) = 0 \tag{3.11}$$

Thus, using only the supersymmetry transformations we have obtained two conditions on $R^{\mu}_{\ \nu}$, (3.7) and (3.11). In turn the condition (3.7) implies

$$J^{A}_{\ C}R^{C}_{\ B} = (\eta_1\eta_2)R^{A}_{\ C}J^{C}_{\ B} \tag{3.12}$$

Assuming that $R^{A}_{\ B}$ is constant (3.11) implies

$$R^{A}_{\ M}f_{BN}^{\ \ \ M} + R^{C}_{\ B}R^{M}_{\ N}f_{CM}^{\ \ \ A} = J^{S}_{\ B}J^{L}_{\ N}\left(R^{A}_{\ M}f_{SL}^{\ \ \ M} + R^{C}_{\ S}R^{M}_{\ L}f_{CM}^{\ \ \ A}\right) \tag{3.13}$$

To better understand the meaning of the conditions (3.12) and (3.13) we choose the basis adapted to the Manin triple $(\mathbf{g}, \mathbf{g}_{-}, \mathbf{g}_{+})$ discussed in the previous section. For B-type conditions, (3.12) implies that $R^{a}_{\ \bar{b}} = R^{\bar{a}}_{\ b} = 0$. Taking this into account we rewrite the condition (3.13) as follows

$$R^{c}_{\ b}R^{m}_{\ n}f_{cm}^{\ \ \ a} + R^{a}_{\ m}f_{bn}^{\ \ \ m} = 0 \tag{3.14}$$

$$R^{\bar{c}}_{\ \bar{b}}R^{\bar{m}}_{\ \bar{n}}f_{\bar{c}\bar{m}}^{\ \ \ \bar{a}} + R^{\bar{a}}_{\ \bar{m}}f_{\bar{b}\bar{n}}^{\ \ \ \bar{m}} = 0. \tag{3.15}$$

We conclude that $R^{a}_{\ b}$ is a Lie algebra automorphism for $\mathbf{g}_{+}$ (more presicely, $[T_a, T_b] = -f_{ab}^{\ \ \ c}T_c$) and that $R^{\bar{a}}_{\ \bar{b}}$ is a Lie algebra automorphism for $\mathbf{g}_{-}$.

For the A-type boundary conditions, (3.12) yields that $R^{a}_{\ b} = R^{\bar{a}}_{\ \bar{b}} = 0$. Taking this into account we rewrite the condition (3.13) as

$$R^{\bar{c}}_{\ b}R^{\bar{m}}_{\ n}f_{\bar{c}\bar{m}}^{\ \ \ \bar{a}} + R^{\bar{a}}_{\ m}f_{bn}^{\ \ \ m} = 0, \tag{3.16}$$

$$R^{c}_{\ \bar{b}}R^{m}_{\ \bar{n}}f_{cm}^{\ \ \ a} + R^{a}_{\ \bar{m}}f_{\bar{b}\bar{n}}^{\ \ \ \bar{m}} = 0. \tag{3.17}$$

We conclude that $R^{\bar{a}}_{\ b}$ is a Lie algebra homomorphism from $\mathbf{g}_{+}$ to $\mathbf{g}_{-}$ and $R^{a}_{\ \bar{b}}$ is a Lie algebra homomorphism from $\mathbf{g}_{-}$ to $\mathbf{g}_{+}$.

We summarize the results. For the B-type supersymmetry we have the following boundary conditions

$$j_-^a = \eta_1 R^a_{\ b} j_+^b, \qquad k_=^a = R^a_{\ b} k_{+\!\!+}^b, \qquad j_-^{\bar a} = \eta_1 R^{\bar a}_{\ \bar b} j_+^{\bar b}, \qquad k_=^{\bar a} = R^{\bar a}_{\ \bar b} k_{+\!\!+}^{\bar b}, \tag{3.18}$$

where $R^a_{\ b}$ is a Lie algebra automorphism for $\mathbf{g}_-$ and $R^{\bar a}_{\ \bar b}$ is a Lie algebra automorphism for $\mathbf{g}_+$. For the A-type supersymmetry the boundary conditions are

$$j_-^a = \eta_1 R^a_{\ \bar b} j_+^{\bar b}, \qquad k_=^a = R^a_{\ \bar b} k_{+\!\!+}^{\bar b}, \qquad j_-^{\bar a} = \eta_1 R^{\bar a}_{\ b} j_+^{b}, \qquad k_=^{\bar a} = R^{\bar a}_{\ b} k_{+\!\!+}^{b}, \tag{3.19}$$

where $R^a_{\ \bar b}$ is a Lie algebra homomorphism from $\mathbf{g}_-$ to $\mathbf{g}_+$ and $R^{\bar a}_{\ b}$ is a Lie algebra homomorphism from $\mathbf{g}_+$ to $\mathbf{g}_-$. It is important to stress that the requirement of conformal invariance does not enter here. In our derivation we used only the supersymmetry transformations and the assumption that $R^A_{\ B}$ is a constant.

In the above derivation we analysed the problem first in terms of X^μ and $\psi^\mu_\pm$ and then expressed the results in terms of the affine currents. Of course we could first rewrite the supersymmetry transformations in terms of the affine currents and then the analyse the boundary condition for the affine currents. The result would not change. For the sake of completeness we also record the supersymmetry transformations in terms of affine currents.

The manifest supersymmetry

$$\delta_1 \mathcal{J}^A_\pm = i\epsilon_1^+ Q_+ \mathcal{J}^A_\pm + i\epsilon_1^- Q_- \mathcal{J}^A_\pm \tag{3.20}$$

with $Q_\pm$ defined in (A.23) can be written in the components as follows

$$\begin{cases} \delta_1 j^A_\pm = -i\epsilon_1^\pm k^A_{\substack{+\!\!+ \\ =}} \\ \delta_1 k^A_{\substack{+\!\!+ \\ =}} = -\epsilon_1^\pm \partial_{\substack{+\!\!+ \\ =}} j^A_\pm \end{cases} \tag{3.21}$$

where we have used the equations of motion. Using the definition (2.13) of N=1 affine currents we can write the nonmanifest supersymmetry (2.2) (on-shell) as

$$\delta_2 \mathcal{J}^A_\pm = -\epsilon_2^\pm J^A_{\ B} D_\pm \mathcal{J}^B_\pm \mp \epsilon_2^\pm f_{MK}^{\ \ \ A} J^K_{\ B} \mathcal{J}^B_\pm \mathcal{J}^M_\pm . \tag{3.22}$$

In components the transformation (3.22) become

$$\begin{cases} \delta_2 j^A_\pm = -i\epsilon_2^\pm J^A_{\ B} k^B_{\substack{+\!\!+ \\ =}} \mp \epsilon_2^\pm f_{MK}^{\ \ \ A} J^K_{\ B} j^B_\pm j^M_\pm \\ \delta_2 k^A_{\substack{+\!\!+ \\ =}} = \epsilon_2^\pm J^A_{\ B} \partial_{\substack{+\!\!+ \\ =}} j^B_\pm \mp \epsilon_2^\pm f_{K[B}^{\ \ \ \ A} J^K_{\ M]} k^M_{\substack{+\!\!+ \\ =}} j^B_{\substack{+\!\!+ \\ =}} \end{cases} \tag{3.23}$$

Now starting from $j^A_- = \eta_1 R^A_{\ B} j^B_+$ and using the transformations (3.21) and (3.23) we easily rederive the previous results (3.12) and (3.13).

In fact the transformations (3.21) and (3.23) can be "complexified" in the Manin basis. Introducing $\delta = \delta_1 + \delta_2$ and $\epsilon^\alpha = \epsilon_1^\alpha + i\epsilon_2^\alpha$ we can write the transformation as follows

$$\begin{cases} \delta j^a_\pm = -i\epsilon^\pm k^a_{\pm\!\!\pm} \pm \frac{1}{2}(\epsilon^\pm - \bar{\epsilon}^\pm) f_{bc}{}^a j^b_\pm j^c_\pm \\ \delta k^a_{\pm\!\!\pm} = -\bar{\epsilon}^\pm \partial_{\pm\!\!\pm} j^a_\pm \pm (\epsilon^\pm - \bar{\epsilon}^\pm) f_{bc}{}^a k^c_{\pm\!\!\pm} j^b_\pm \end{cases} \tag{3.24}$$

(with a similar expression for the $\bar{a}$-part).

4 N=2 supreconformal boundary conditions

In this section we incorporate the requirement of conformal invariance into the boundary conditions. We derive the N=2 superconformal boundary conditions by imposing appropriate boundary conditions on the conserved (2,2) currents ($T_{\pm\pm}$, $G^1_\pm$, $G^2_\pm$, $J_\pm$). The N=1 superfield and component forms of these currents can be found in [5]. Here we present them in terms of the fermionic and bosonic affine currents, $j^A_\pm$ and $k^A_{\pm\!\!\pm}$. Using results from [5] and the definitions (2.14) it is a straightforward exercise to write the N=2 currents as

$$T_{++} = k^A_{+\!\!+} \eta_{AB} k^B_{+\!\!+} + i j^A_+ \eta_{AB} \partial_{+\!\!+} j^B_+ + i k^A_{+\!\!+} j^B_+ j^C_+ f_{ABC}, \tag{4.1}$$

$$T_{--} = k^A_{=} \eta_{AB} k^B_{=} + i j^A_- \eta_{AB} \partial_{=} j^B_- - i k^A_{=} j^B_- j^C_- f_{ABC}, \tag{4.2}$$

$$G^1_+ = j^A_+ \eta_{AB} k^B_{+\!\!+} + \frac{i}{3} j^A_+ j^B_+ j^C_+ f_{ABC}, \tag{4.3}$$

$$G^1_- = j^A_- \eta_{AB} k^B_{=} - \frac{i}{3} j^A_- j^B_- j^C_- f_{ABC}, \tag{4.4}$$

$$G^2_+ = j^A_+ J_{AB} k^B_{+\!\!+}, \tag{4.5}$$

$$G^2_- = j^A_- J_{AB} k^B_{=}, \tag{4.6}$$

$$J_+ = j^A_+ j^B_+ J_{AB}, \qquad J_- = j^A_- j^B_- J_{AB}. \tag{4.7}$$

We define the following linear combinations of $G^i_\pm$

$$\mathcal{G}_\pm = \frac{1}{2}(G^1_\pm + iG^2_\pm), \qquad \bar{\mathcal{G}}_\pm = \frac{1}{2}(G^1_\pm - iG^2_\pm) \tag{4.8}$$

Using the properties of Manin triple $(\mathbf{g}, \mathbf{g}_-, \mathbf{g}_+)$ we write $\mathcal{G}_\pm$ and $\bar{\mathcal{G}}_\pm$ as follows

$$\mathcal{G}_\pm = j^a_\pm \eta_{a\bar{b}} k^{\bar{b}}_{\pm\!\!\pm} \pm \frac{i}{2} j^a_\pm j^{\bar{b}}_\pm j^{\bar{c}}_\pm f_{a\bar{b}\bar{c}} \pm \frac{i}{2} j^a_\pm j^b_\pm j^{\bar{c}}_\pm f_{ab\bar{c}}, \tag{4.9}$$

$$\bar{\mathcal{G}}_\pm = j^{\bar{a}}_\pm \eta_{\bar{a}b} k^b_{\pm\!\!\pm} \mp \frac{i}{2} j^a_\pm j^b_\pm j^{\bar{c}}_\pm f_{ab\bar{c}} \mp \frac{i}{2} j^a_\pm j^{\bar{b}}_\pm j^{\bar{c}}_\pm f_{a\bar{b}\bar{c}}. \tag{4.10}$$

We see that once we have a Lie algebra $\mathbf{g}$ with invariant inner product η (given by η_{AB}) and a Manin triple $(\mathbf{g}, \mathbf{g}_-, \mathbf{g}_+)$ defined with respect to η we may define the N=2 currents (4.1), (4.2), (4.7), (4.9) and (4.10). In fact they will obey the correct N=2 alegbra[4], [14] and [15].

[4]One should keep in mind that there are two sets of bosonic affine currents which differ by the two-fermion term.

To ensure N=2 superconformal symmetry on the boundary we have to impose the following conditions[5] on the currents (4.1)–(4.7),

$$T_{++} - T_{--} = 0, \qquad G^1_+ - \eta_1 G^1_- = 0, \tag{4.11}$$

$$G^2_+ - \eta_2 G^2_- = 0, \qquad J_+ - (\eta_1\eta_2) J_- = 0. \tag{4.12}$$

The conditions (4.11) ensure N=1 superconformal invariance. Starting from the ansatz $j^A_- = \eta_1 R^A{}_B j^B_+$ with constant $R^A{}_B$ and solving the conditions (4.11) we derive the bosonic counterpart

$$k^A_= = R^A{}_B k^B_{\#}, \tag{4.13}$$

together with the additional properties

$$R^C{}_A \eta_{CD} R^D{}_B = \eta_{AB}, \qquad f_{ABC} + R^L{}_A R^M{}_B R^N{}_C f_{LMN} = 0 \tag{4.14}$$

Thus N=1 superconformal invarince implies that $R^A{}_B$ should be a Lie algebra automorphism. Solving the conditions (4.12) we arrive at the condition

$$J^A{}_C R^C{}_B = (\eta_1\eta_2) R^A{}_C J^C{}_B. \tag{4.15}$$

As one would expect the conditions (4.14) and (4.15) are stronger than the conditions (3.12) and (3.13) which come from the N=2 supersymmetry alone. The difference is the condition $R^C{}_A \eta_{CD} R^D{}_B = \eta_{AB}$. Thus adding this condition to (3.12) and (3.13) we recover (4.14) and (4.15).

The conserved currents $J_\pm$ generate two R-rotations which act trivially on the bosonic fields but non-trivially on the fermions. Because of the boundary condition $J_+ - (\eta_1\eta_2)J_- = 0$ only one combination of these R-rotations survives as a symmetry in the presence of a boundary. Thus for the B-type we have the following R-symmetry

$$\begin{cases} j^A_+ \rightarrow \cos\alpha \; j^A_+ + \sin\alpha \; J^A{}_B j^B_+ \\ j^A_- \rightarrow \cos\alpha \; j^A_- + \sin\alpha \; J^A{}_B j^B_- \end{cases} \tag{4.16}$$

and for the A-type

$$\begin{cases} j^A_+ \rightarrow \cos\alpha \; j^A_+ + \sin\alpha \; J^A{}_B j^B_+ \\ j^A_- \rightarrow \cos\alpha \; j^A_- - \sin\alpha \; J^A{}_B j^B_- \end{cases} \tag{4.17}$$

In the Manin basis these rotations are ($j^a_\pm \rightarrow e^{i\alpha} j^a_\pm$, $j^{\bar{a}}_\pm \rightarrow e^{-i\alpha} j^{\bar{a}}_\pm$) and ($j^a_\pm \rightarrow e^{\pm i\alpha} j^a_\pm$, $j^{\bar{a}}_\pm \rightarrow e^{\mp i\alpha} j^{\bar{a}}_\pm$) respectively.

[5]Classically these conditions make sense only on-shell since the currents are defined modulo the equations of motion.

5 Summary and discussion

We consider a WZW model defined over a connected Lie group $\mathcal{G}$, such such that its Lie algebra $\mathbf{g}$ comes equipped with a symmetric ad-invariant nondegenerate bilinear form $(T_A, T_B) = \eta_{AB}$ and can be decomposed into a pair of maximally isotropic subalgebras $\mathbf{g}_-$, $\mathbf{g}_+$ with respect to η and $\mathbf{g}$ as a vector space is the direct sum of $\mathbf{g}_-$ and $\mathbf{g}_+$. This ordered triple of algebras $(\mathbf{g}, \mathbf{g}_-, \mathbf{g}_+)$ is called a Manin triple. It is easy to see that the dimensions of sublagebras $\mathbf{g}_-$, $\mathbf{g}_+$ are equal and that the bases $\{T_a\}$, $\{T^a\}$ may be choosen such that

$$(T_a, T_b) = 0, \qquad (T_a, T^b) = \delta^b_{\ a}, \qquad (T^a, T^b) = 0. \tag{5.18}$$

The algebraic structure of $\mathbf{g}$ is completely determined by the structures of the maximal isotropic subalgebras in the basis (5.18)

$$[T_a, T_b] = f_{ab}{}^c T_c, \qquad [T^a, T^b] = f^{ab}{}_c T^c, \qquad [T_a, T^b] = f_{ca}{}^b T^c + f^{bc}{}_a T_c. \tag{5.19}$$

Introducing the corresponding affine fermionic and bosonic currents $(j_{\pm a}, k_{\underline{\underline{+}} a})$ and $(j^a_{\pm}, k^a_{\underline{\underline{+}}})$ we construct the N=(2,2) currents ($T_{\pm\pm}$, $G^1_{\pm}$, $G^2_{\pm}$, $J_{\pm}$) as defined in (4.1), (4.2), (4.7), (4.9) and (4.10) correspondently. In the presence of a boundary, appropriate gluing conditions should be imposed on the affine currents. There are two ways of gluing the fermionic currents: the first is

$$j^a_- = \eta_1 R^a_{\ b} j^b_+, \tag{5.20}$$

which we call the B-type and the second is

$$j^a_- = \eta_1 R^{ab} j_{+b}, \tag{5.21}$$

which we call the A-type. To preserve half of the bulk supersymmetry the gluing matrix should satisfy the following: $R^a_{\ b} \in Aut(\mathbf{g}_-)$ for the B-type and $R^{ab} \in Hom(\mathbf{g}_-, \mathbf{g}_+)$ for the A-type. Gluing conditions must also be imposed on the bosonic affine currents. In addition conformal invariance requires R to preserve the form η. Therefore, for both A- and B-types, superconformal boundary conditions require $R \in Aut(\mathbf{g})$.

It is easy to give the example of the B-type brane with $R = I$ which would correspond the branes localized along conjugacy classes [16]. However it is a bit problematic to give a simple example of the A-type brane since it would depend on the definition of the form η. We hope to come back to these examples elsewhere.

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A (1,1) supersymmetry

In this we collect our conventions on the N=1 supersymmetry which we use through the text.

We deal with real (Majorana) two-component spinors $\psi^\alpha = (\psi^+, \psi^-)$. Spinor indices are raised and lowered by the second-rank antisymmetric symbol $C_{\alpha\beta}$, which defines the spinor inner product:

$$C_{\alpha\beta} = -C_{\beta\alpha} = -C^{\alpha\beta}, \quad C_{+-} = i, \quad \psi_\alpha = \psi^\beta C_{\beta\alpha}, \quad \psi^\alpha = C^{\alpha\beta}\psi_\beta. \tag{A.22}$$

Throughout the paper we use $(\pm\!\!\!+, =)$ as worldsheet indices, and $(+,-)$ as two-dimensional spinor indices. We also use superspace conventions where the pair of spinor coordinates of the two-dimensional superspace are labelled $\theta^\pm$, and the covariant derivatives $D_\pm$ and supersymmetry generators $Q_\pm$ satisfy

$$\begin{aligned} D_+^2 &= i\partial_{+\!\!\!+}, \quad D_-^2 = i\partial_= \quad \{D_+, D_-\} = 0 \\ Q_\pm &= iD_\pm + 2\theta^\pm \partial_{\substack{+\!\!\!+ \\ =}} \end{aligned} \tag{A.23}$$

where $\partial_{\substack{+\!\!\!+ \\ =}} = \partial_0 \pm \partial_1$. In terms of the covariant derivatives, a supersymmetry transformation of a superfield Φ is then given by

$$\begin{aligned} \delta\Phi &\equiv i(\varepsilon^+ Q_+ + \varepsilon^- Q_-)\Phi \\ &= -(\varepsilon^+ D_+ + \varepsilon^- D_-)\Phi + 2i(\varepsilon^+\theta^+\partial_{+\!\!\!+} + \varepsilon^-\theta^-\partial_=)\Phi. \end{aligned} \tag{A.24}$$

The components of a superfield Φ are defined via projections as follows,

$$\Phi| \equiv X, \quad D_\pm\Phi| \equiv \psi_\pm, \quad D_+D_-\Phi| \equiv F_{+-}, \tag{A.25}$$

where a vertical bar denotes "the $\theta = 0$ part of ". Thus, in components, the $(1,1)$ supersymmetry transformations are given by

$$\begin{cases} \delta X^\mu = -\epsilon^+\psi_+^\mu - \epsilon^-\psi_-^\mu \\ \delta\psi_+^\mu = -i\epsilon^+\partial_{+\!\!\!+}X^\mu + \epsilon^- F_{+-}^\mu \\ \delta\psi_-^\mu = -i\epsilon^-\partial_= X^\mu - \epsilon^+ F_{+-}^\mu \\ \delta F_{+-}^\mu = -i\epsilon^+\partial_{+\!\!\!+}\psi_-^\mu + i\epsilon^-\partial_-\psi_+^\mu \end{cases} \tag{A.26}$$

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